Algorithmic Analysis of a Model Checking Partial Order Reduction Scheme Using Alloy

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Subodh Sharma's Class
Anywhere
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  - Peled, CAV 1994; FMSD 1996
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- particularly effective for concurrent systems
  - multithreaded programs, ...
- many robust, widely-used model checking tools
  - SPIN — has its own input language, PROMELA, highly optimized
  - Divine (C++), CIVL (C+...), NuSMV, Java PathFinder (Java), ...

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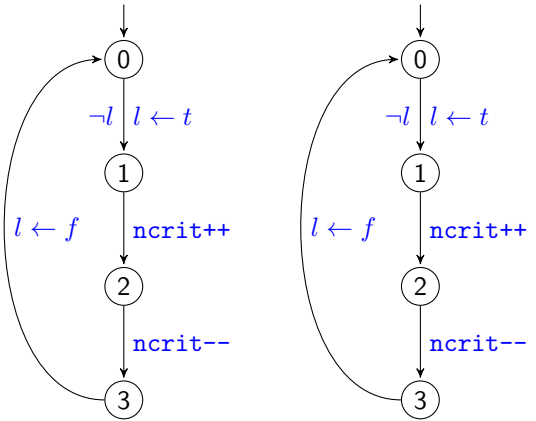
   - temporal logic, automata, ...
3. apply automated algorithmic (graph, automata, ...) techniques to check that all executions of the program satisfy the properties
   - typically: explore the reachable states of the program
   - simultaneously updating a property automaton
   - report a violation as soon as an accepting cycle is detected

Example: mutual exclusion: Promela program

```
bool lock = false;
int ncrit = 0;

active [2] proctype thread() {
  do
  :: atomic { !lock -> lock = true }
     ncrit++;
     // ...this is the critical section...
     ncrit--;
     lock = false
  od
}
```

Example: mutual exclusion: state-transition system



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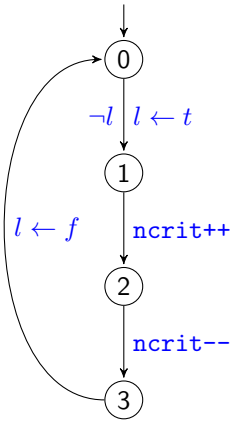
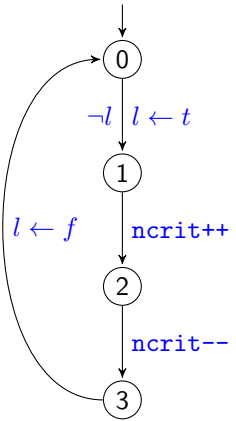
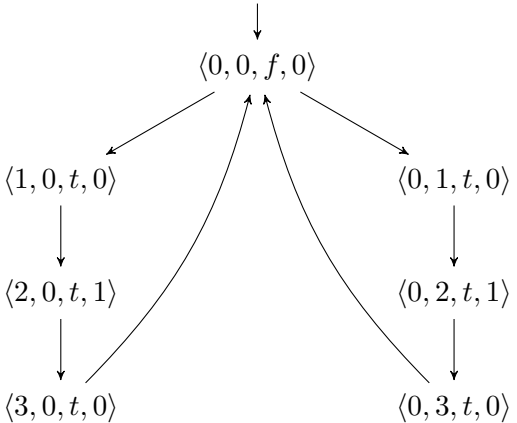


Thread 0



Thread 1



States: $\langle \mathsf{pc}_0, \mathsf{pc}_1, l, \mathtt{ncrit} \rangle$

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- Büchi automaton (negated) ...

Büchi automata

A Büchi automaton is a tuple

$$\mathcal{B} = \langle S, \Delta, \Sigma, \delta, F \rangle,$$

where

1. S is a finite set of *automaton states*
2. $\Delta \subseteq S$ is the set of *initial states*
3. Σ is a finite set called the *alphabet*
4. $\delta \subseteq S \times \Sigma \times S$ is the *transition relation*
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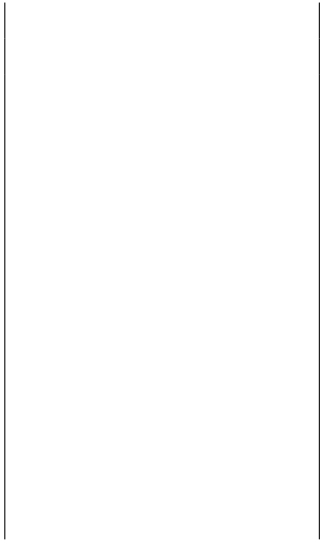
The language of $\mathcal{B}$, denoted $\mathcal{L}(B)$, is the set of all $\xi \in \Sigma^{\omega}$ generated by infinite paths in $\mathcal{B}$ that pass through an accepting state infinitely often.

Examples of Büchi automata, $\Sigma = \{a, b, c\}$



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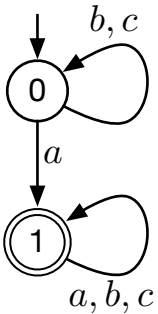

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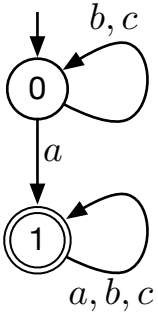

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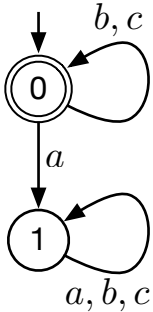

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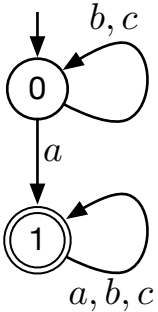

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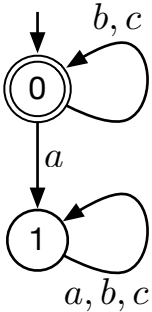
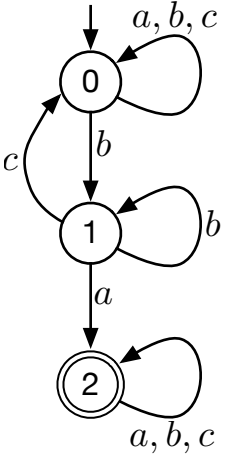


"after some b, a occurs before c"



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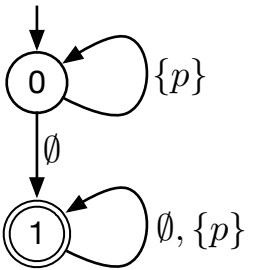
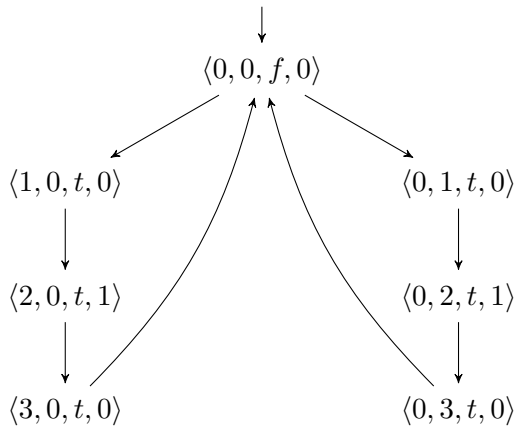
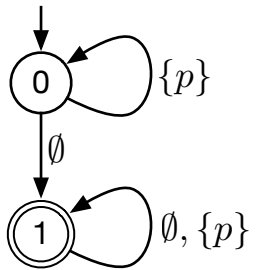
- the BA specifies which infinite words violate the property
- example: a trace violating "always $\texttt{ncrit} \leq 1$":



Automated technique: compute the sycnchronous product



$\otimes$



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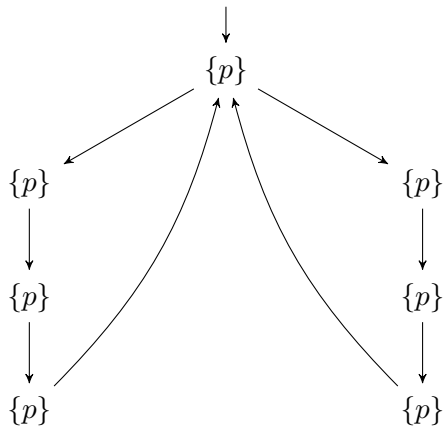

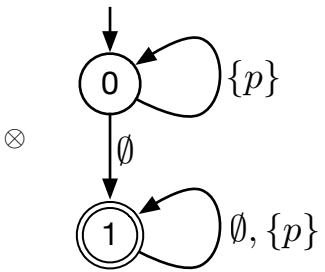

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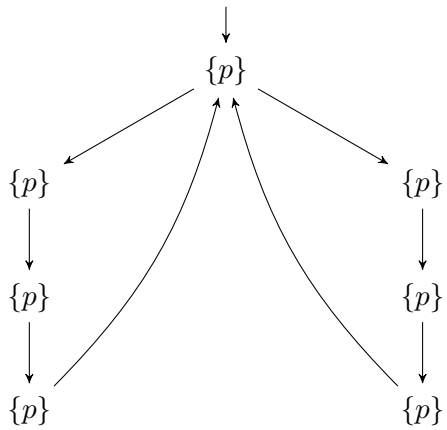

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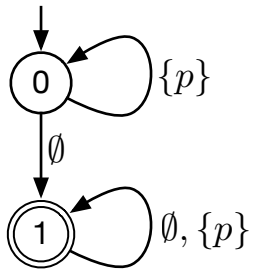

- in the synchronous product, no accepting cycle is reached – try it in SPIN!

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- all properties expressible with LTL operators **F**, **G**, **U** are stutter invariant
- almost any property you would ever want to express is stutter invariant

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  - the order of the two actions doesn't matter
  - you end up in the same state either way
  - if the local action α can not change the value of any atomic proposition...
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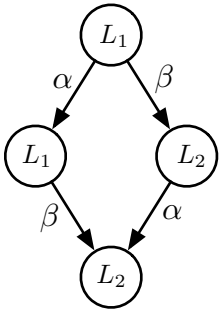

- one path yields trace $\cdots L_1 L_1 L_2 \cdots$
- other path yields trace $\cdots L_1 L_2 L_2 \cdots$
- stutter equivalent!
- either both paths satisfy the property or both violate

Definitions: Programs

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- a set Q of states
- an initial state $\iota \in Q$
- a set T of operations
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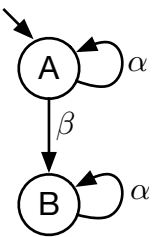

$$Q = \{A, B\}$$
$$\iota = A$$
$$T = \{\alpha, \beta\}$$
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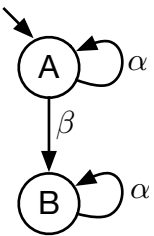

$$
\begin{aligned}
Q &= \{A, B\} \\
\iota &= A \\
T &= \{\alpha, \beta\} \\
\mathsf{en}_\alpha &= \{A, B\} \\
\mathsf{en}_\beta &= \{A\}
\end{aligned}
$$

- for $q \in Q$, define $\mathsf{en}(q) = \{\alpha \in T \mid q \in \mathsf{en}_\alpha\}$

Program Traces

- let AP be a set of atomic propositions
- an interpretation is a function $L\colon Q \to \Sigma = 2^{\mathsf{AP}}$

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- an interpretation is a function $L\colon Q \to \Sigma = 2^{\mathsf{AP}}$
- $\mathcal{L}(P) \subseteq \Sigma^{\omega}$ is the set of infinite traces

$$L(q_0)L(q_1)\cdots$$

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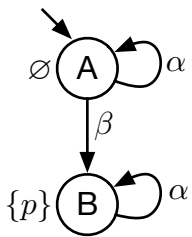

$$\mathsf{AP} = \{p\}$$
$$L(A) = \varnothing$$
$$L(B) = \{p\}$$
$$\varnothing\varnothing\varnothing\{p\}^{\omega} \in \mathcal{L}(P)$$

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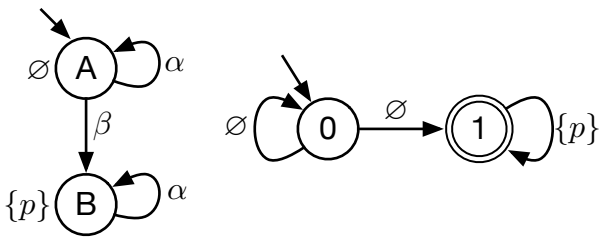

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Given program $P = (Q, T, \iota)$ and Büchi automaton $\mathcal{B}$ with states S:
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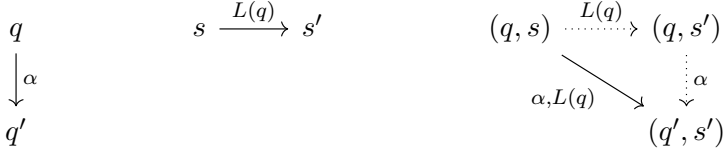

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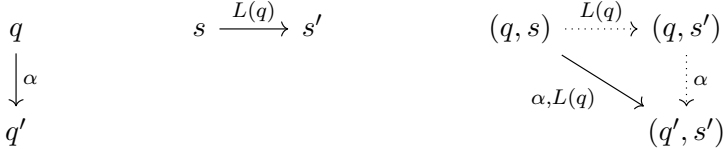

$$\mathcal{L}(P \otimes \mathcal{B}) = \mathcal{L}(P) \cap \mathcal{L}(\mathcal{B})$$

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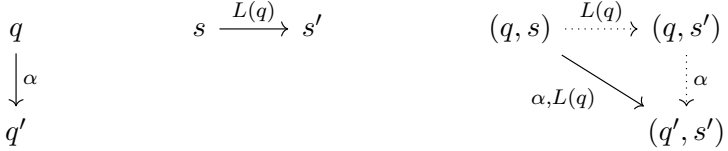

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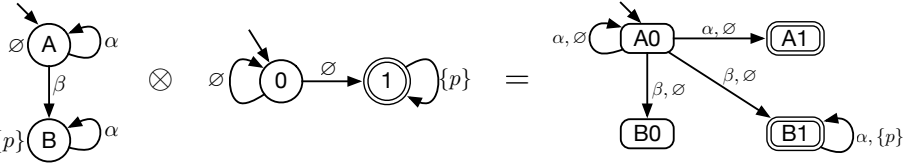

Partial Order Reduction

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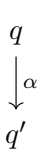
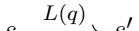
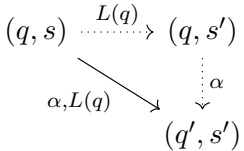






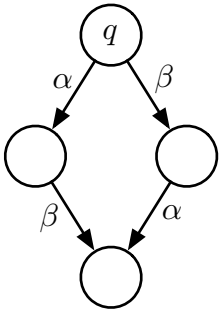
$$\alpha \in \mathsf{amp}(q, s')$$

Dependence and visibility

Definition

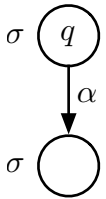
Two operations $\alpha, \beta \in T$ are independent if for all $q \in \mathsf{en}_\alpha \cap \mathsf{en}_\beta$:

1. $\alpha(q) \in \mathsf{en}_\beta$
2. $\beta(q) \in \mathsf{en}_\alpha$ and
3. $\alpha(\beta(q)) = \beta(\alpha(q))$.



Definition

An operation $\alpha \in T$ is invisible if, for all $q \in \mathsf{en}_\alpha$: $L(q) = L(\alpha(q))$.



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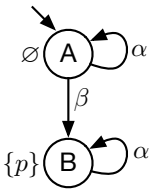
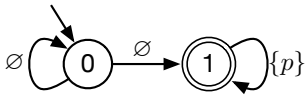
Combined Theorem:

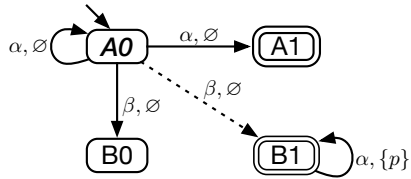
If $\mathcal{L}(\mathcal{B})$ is stutter-invariant and **C0**–**C3** hold, then $\mathcal{L}(A') = \varnothing \Leftrightarrow \mathcal{L}(A) = \varnothing$.

Counterexample to combined theorem





amp	0	1
A	$\{\alpha, \beta\}$	$\{\alpha\}$
B	$\{\alpha\}$	$\{\alpha\}$



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Spin

```
bit p = 0;
active proctype p0() { p=1 }
active proctype p1() { bit x=0; do :: x=0 od }
never {
  B0: do :: !p :: !p -> break od
  accept_B1: do :: p od
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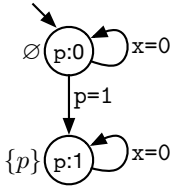
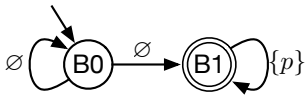
```





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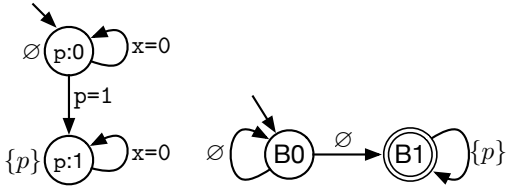


```
$ spin -a test1.pml; cc -o pan -DNOREDUCE pan.c; ./pan -a -n
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```

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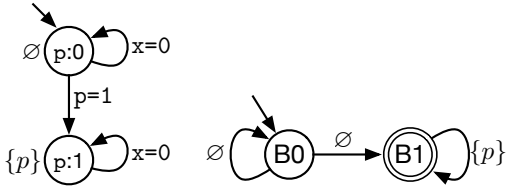


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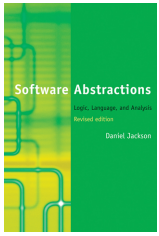
$ spin -a test1.pml; cc -o pan pan.c; ./pan -a -n
warning: for p.o. reduction to be valid the never claim must be stutter-invariant
(never claims generated from LTL formulae are stutter-invariant)
+ Partial Order Reduction
State-vector 36 byte, depth reached 2, errors: 0
        3 states, stored
        2 states, matched
        5 transitions (= stored+matched)
```

Finding Counterexamples to POR Claims

Alloy

- specification language based on relational/first order logic
- a model specifies "signatures" (sets), relations, constraints



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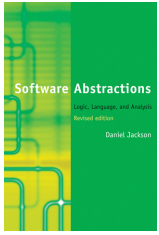
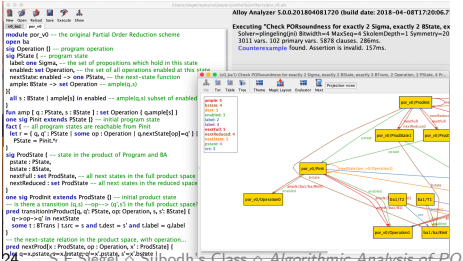


The Alloy Analyzer



given

- an Alloy model
- an assertion
- an upper bound on the size of each signature

produces either

- "assertion may hold"
  - it holds within the bounds, or
- "assertion is violated"

Finding Counterexamples to POR Claims, cont.

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```
fact C1 {
  all q: PState, s: BState | let A=amp[q,s] |
    let r = { q1, q2: PState | some op: Operation-A |
              q1->op->q2 in nextState } |
      all q': q.*r, op1: q'.enabled-A, op2: A | independent[op1, op2]
}
```

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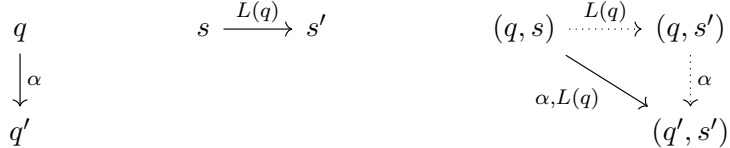

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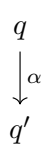

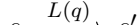

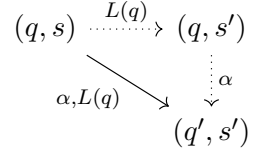

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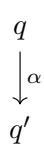

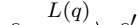

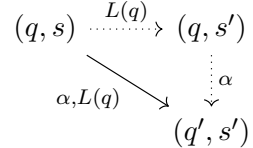

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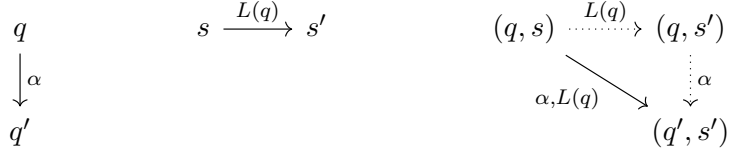
- minor changes to the Alloy model

Variations on the Combined Theorem

- make the ample set a function of the source product state
  - not of the intermediate state

Define $\mathcal{A}'$:



$$\alpha \in \mathsf{amp}(q, s)$$

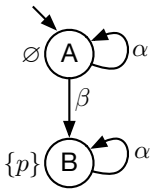
C0–C2: (same)

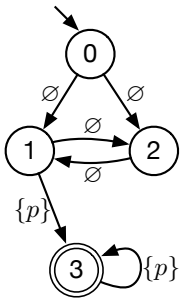
C3: In any cycle in $\mathcal{A}'$:

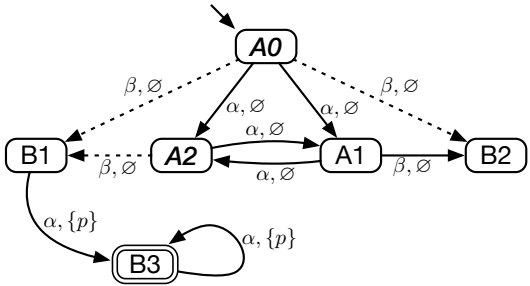
- there is a state $\langle q, s \rangle$ for which $\mathsf{amp}(q, s) = \mathsf{en}(q)$

- minor changes to the Alloy model
- Alloy Analyzer: previous Büchi automaton now OK!

New counterexample







amp	0	1	2	3
A	$\{\alpha\}$	$\{\alpha, \beta\}$	$\{\alpha\}$	$\{\alpha, \beta\}$
B	$\{\alpha\}$	$\{\alpha\}$	$\{\alpha\}$	$\{\alpha\}$

Note: only $A0$ and $A2$ are not fully enabled

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  - Etessami 1999 (Muller automata)
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Consequence of normal form:

1. If $s_1 \xrightarrow{a} s_2 \xrightarrow{b} s_3$ is a path in $\mathcal{B}$, then for some $s_2' \in S$, $s_1 \xrightarrow{a} s_2 \xrightarrow{a} s_2' \xrightarrow{b} s_3$ is a path in $\mathcal{B}$.
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- this is all you need to make on-the-fly POR work

A Fix: Stutter-Invariant Normal Form

Theorem

If $\mathcal{B}$ is in SI normal form and C0–C3 holds, then $\mathcal{L}(A') = \varnothing \Leftrightarrow \mathcal{L}(A) = \varnothing$.

- a (LaTeX) proof is in the paper
- proof goes through for original and variant schemes
- verified for Alloy for small scope [thanks to reviewers]
  - ≤ 3 symbols in alphabet Σ
  - ≤ 4 states in the Büchi automaton
  - ≤ 6 transitions in Büchi automaton
  - ≤ 3 operations in program
  - ≤ 4 program states
  - ≤ 16 product states
  - time: 2,265 seconds

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5. perhaps the best approach
   - use Alloy to prototype (quick iterations)
   - once Alloy says everything is OK, invest in a full mechanized proof