

COL 100 - Lec 12

1. Currying

- Eg: $\overbrace{\text{add } x \ y}^{\text{Definition}} = x + y$
 $\text{fn}: \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$

- Note partial evaluation possible
i.e. $\text{add } 3$ will evaluate to a function
that adds 3 to its argument

$$\text{add } 3 = \text{fn}: \mathbb{Z} \rightarrow \mathbb{Z}$$
$$y \rightarrow y + 3$$

- Where is currying critical?
 - where you expect the output to be a function too!

Eg: function derivatives, ...

Thus

Higher Order function we saw in the last class

→ $\text{HOSumProd} : \text{Op} \rightarrow \text{termFnc} \rightarrow \text{nextfnc} \rightarrow \text{id} \rightarrow \text{L} \rightarrow \text{U}$

$= \begin{cases} \text{id} & \text{if } L > U \\ \text{else} & \text{Op (termFnc L) (HOSumProd} \end{cases}$

$\text{Op termFnc nextfnc}$
 $\text{id} (\text{nextfnc L}) \text{ U})$

Could also be specified as
the following:

*** Show fact Computation!*
*** Show sum Computation!*

→ $\text{HOSumProd} (\text{Op}, \text{termFnc}, \text{nextfnc}, \text{id}, \text{L}, \text{U})$

$= \begin{cases} \text{id} & \text{if } (L > U) \\ \text{else} & \text{Op (termfnc L), HOSumProd (} \dots \text{)} \end{cases}$

$\text{HOSumProd} (\dots) \text{ U})$

↳ in the non-curried form

What have we looked so far

- Procedures as arguments
(or functions)

Let us look now at constructing procedures
as λ expressions.
(lambda)

Motivation: In the example to compute
sum-series-pi we needed

- pi-term to be $(1.0 / x * (x + 2))$
- pi-next to be $(x + 4)$

It is awkward to define trivial procedures
pi-term & pi-next just so we can define our higher
order function.

A more convenient way is to use lambda expressions

Mathematically,

$\lambda x. (x+4)$

formal parameters body

SML

fn x => (x+4)

The resulting procedure is the same as what one would have obtained using keyword fun with a procedure name, except that lambda procedures are no-name (or nameless) procedures

Procedures as general methods

Elaborate Example

- finding roots of equations by the half-interval method

$$f(x) = 0 \quad [f \text{ is continuous}]$$

idea is if we are given pts a, b

s.t. $f(a) < 0 < f(b)$

then f must have one root (or zero)

Detⁿ a & b .

take

$$x = \frac{a+b}{2}$$

• if $f(x) > 0$ then the root is
between a & x

• if $f(x) < 0$ then the root is
between x & b

• Continuing this way we reduce the
interval by half at each step until
some tolerance level is reached
(let us call that T)

• Then, the process stops in $O(\log(L/T))$ steps.

[L is the length of original interval
 L/T is the tolerance]

• function (or procedure) to implement our strategy

fun search f neg-pt pos-pt =

let

val midpt = average neg-pt pos-pt

in

if (close-enough? neg-pt pos-pt)
then midpt

Else

let

val test-value = f midpt

in

if test-value > 0.0 then

search f neg-pt midpt

Else if test-value < 0.0 then

search f midpt pos-pt

Else midpt

fun close-enough? x y
= (abs (x - y) < 0.001)


```
fun half-interval f a b =
```

```
  let
```

```
    val a-v = f a
```

```
    val b-v = f b
```

```
  in
```

```
    if a-v < 0.0 andalso b-v ≥ 0.0 then
```

```
      search f a b
```

```
  else if a-v ≥ 0.0 andalso b-v < 0.0 then
```

```
    search f b a
```

```
  else
```

```
    raise Not Opposite Signs
```

half-interval-method sin 2.0 4.0

for $x^3 - 2x - 3 = 0$, betⁿ 1 2 2

half-interval-method (fn x $\Rightarrow x * x * x - 2 * x - 3$) 1.0 2.0